Natural Maths - Mandelbrot Set

Formal Definition of the Mandelbrot set within Natural Maths

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Introduction

The classical Mandelbrot set arises from iterating a complex quadratic map $z \mapsto z^2 + c$. Its structure fundamentally depends on complex rotation: the imaginary axis is built into the dynamics.

Here we introduce a *real* analogue consistent with the axioms of **Natural Maths**, a framework in which only the two curvature orientations ± 1 exist and imaginary rotation is not permitted.

Under these constraints the unique admissible quadratic map is:

$$x_{n+1} = \sigma_n x_n^2 + c, \qquad \sigma_n \in \{-1, +1\}$$

A key innovation is the curvature-flip operator:

$$\sigma_{n+1} = \sigma_n \quad \text{unless } |x_{n+1}| > 1 + |b|\kappa, \text{ in which case } \sigma_{n+1} = -\sigma_n$$

This creates a 2-dimensional parameter space (c, b) on $\mathbb{R}$, where the vertical axis encodes initial curvature bias, replacing the imaginary axis of $\mathbb{C}$.

The resulting object, the **Natural-Maths Mandelbrot Set**, exhibits both familiar features (period-doubling cascades, stability windows, chaotic bands) and new phenomena not present in complex dynamics (diagonal resonance boundaries, barcode escape spectra, $\varkappa$-dependent curvature layers).

For $\kappa = 0$, the fractal collapses into a remarkably clean **discrete spectrum**, which provides a window into the stability structure of dynamics. The **curvature-sensitivity parameter** κ links this dynamics to the **unified gravity field** of the ambient physical reality.

Formal Definition

Let

- $c \in \mathbb{R}$ be a real parameter,
- $b \in \mathbb{R}$ be an initial curvature-bias,
- $\kappa \in \mathbb{R}$ be a curvature-sensitivity constant,
- $x_0 = b$
- $\sigma_n \in \{-1, +1\}$ be a curvature orientation at step n

Define the curvature-flip operator:

$$\sigma_{n+1} = \begin{cases} -\sigma_n, & \text{if } |\, x_{n+1} \,| > 1 + |b|\kappa, \\ [4pt]\ \sigma_n, & \text{otherwise}. \end{cases}$$

Define the Natural-Maths quadratic map:

$$x_{n+1} = \sigma_n x_n^2 + c$$

Natural-Maths Mandelbrot Set

The parameter pair (c, b) belongs to the Natural-Maths Mandelbrot Set if and only if the orbit $\{x_n\}_{n\geq 0}$ remains bounded:

$$(c, b) \in \mathscr{M}_{\mathrm{NM}}(\kappa) \quad \Longleftrightarrow \quad \sup_n |x_n| < \infty$$

And is visualised by scanning:

- **horizontal axis** → parameter c
- **vertical axis** → initial bias $b = x_0$
- **color** → escape speed (iteration count)

This produces the "barcode spectrum" and its ϰ-deformation family.

2. NOTATION STANDARD

We adopt the following compact notation:

Curvature orientation

$\sigma_n \in \{-1, +1\}$

Curvature-flip threshold

$T(b, \kappa) = 1 + |b| \kappa$

Curvature-flip operator

$\sigma_{n+1} = \sigma_n (-1)^{\mathbf{1}_{\{|x_{n+1}|>T\}}}$

Natural-Maths quadratic map

$f_\sigma(x, c) = \sigma x^2 + c$

Orbit

$x_{n+1} = f_{\sigma_n}(x_n, c), \quad x_0 = b$

Natural-Maths Mandelbrot family

$\mathscr{M}_{\mathrm{NM}}(\kappa) = \{(c, b) \in \mathbb{R}^2 : \sup_n |x_n| < \infty\}$

ϰ-slice

$\mathscr{M}_{\mathrm{NM}}(\kappa)\Big|_c$ is a 1D bifurcation spectrum over fixed c

3.1 Absence of the imaginary axis

Classical Mandelbrot dynamics rely fundamentally on *complex rotation*:

$z \mapsto z^2 + c$ where $z \in \mathbb{C}$

Natural-Maths rejects the imaginary axis entirely. The only allowable curvature states are:

- $\sigma = +1$: curvature-preserving,
- $\sigma = -1$: curvature-reversing.

Thus the only admissible quadratic iteration consistent with the axioms is:

$$x_{n+1} = \sigma_n x_n^2 + c$$

This is the unique analogue of the complex quadratic map in Natural-Maths.

3.2 Introduction of a curvature-flip operator

The essential innovation is the **threshold-driven sign flip**:

$$|x| > 1 + |b|\kappa \quad \Rightarrow \quad \sigma \to -\sigma$$

This creates:

- discrete curvature phases,
- diagonal resonance boundaries,
- "barcode" escape spectra,
- bifurcation cascades richer than Feigenbaum's,
- symmetry patterns not present in $\mathbb{C}$.

This mechanism **has no analogue in classical dynamical systems**.

3.3 A 2-parameter Mandelbrot family on $\mathbb{R}^2$

Classical Mandelbrot uses:

- real axis → real part of c
- imaginary axis → imaginary part of c

Natural-Maths replaces "imaginary direction" with **initial curvature bias** *b*

Thus instead of $\mathbb{C}$, we get a new real dynamical plane:

$$(c, b) \in \mathbb{R}^2$$

This shift breaks the symmetry structures of $\mathbb{C}$ and produces entirely new phenomenology.

3.4 Emergence of a discrete curvature spectrum

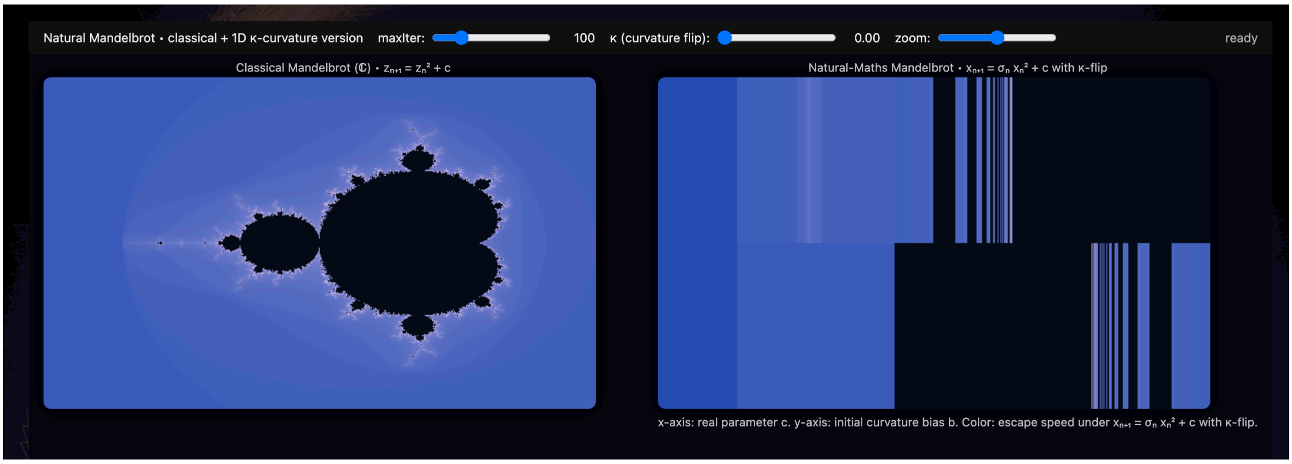
At $\kappa = 0$:

- the system becomes piecewise-quadratic,
- orientation is constant,
- the fractal collapses into vertical resonance bands,
- producing a **clean, low-noise spectrum**.

This "spectrum view" is invisible in the traditional Mandelbrot set and appears to encode:

- stability windows,
- bifurcation cascades,
- discrete curvature resonances.

It may be more diagnostically useful than the classical fractal boundary.



Under the Natural Maths axioms, this is the *unique* quadratic iteration map. No alternative Mandelbrot formulation exists in Natural Maths.